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B.Tech (Sem.2nd)

ENGINEERING. MATHEMATICS-II
Subject Code :BTAM-102
Paper ID : [A1111]

Time: 3 Hrs. **Max. Marks :60**

Note:- (1) Section-A is compulsory. Attempt any five question from section B and Section-C Selecting at east two from each section. Each question of section B and C Carry eight marks.

SECTION-A (2x10=20)

Q1. (a) Test whether the set {(1,1,1,),(1,1,0),(1,0,1)} of vectors is Linearly independent or dependent.

(b) Prove that the eigen values of unitary matrix are of unit modulus.

(c) Define the logarithmic function of a complex variable and hence find the general value of *log* (-i).

(d) Express $\text{Sin}^5\theta\cos^2\theta$ in a series of sines of multiples of θ.

(e) Discuss the convergence/divergence of the series $\sum_{n=2}^{\infty}\frac{\cos n\pi}{n\sqrt{n}}$

(f) Find the general solution of the equation $\frac{dy}{dx} = \sin\left(y - x\frac{dy}{dx}\right)$.

(g) Obtain the particular solution of the equation $\frac{d^2y}{dt^2}$ +4y=cos 2t.

(h) Define Hermetian and skew-hermetian matrix with one example of each.

(i) For what value of "k" the differential equation $xy^3dx+kx^2y^2dy=0$ is an exact equation.

(j) State Integral test and use it to test the convergence/divergence of the series

$$\sum_{n=2}^{\infty} \frac{1}{n\log n}$$

SECTION-B

2x10=20

Q2. (a) Solve the following simultaneous differential equation (5)

$$\frac{dx}{dt} + 3y + 4x = t, \frac{dy}{dt} + 2x + 5y = e^t$$

(b) Find the particular solution of the differential equation $y'' - 4y' + 3y = e^x \cos 2x$ by using operator method. (3)

Q3. (a) Use method of variation of parameters to find the general solution of the differential equation $y'' + 3y' + 2y = 2e^x$. (4)

(b) Find the complete solution of the differential equation (4)

$$(3x+2)^2 y'' + 3(3x+2)y' - 36y = 3x^2 + 4x + 1$$

Q4. (a) An e.m.f $E_0 \sin pt$ is applied at t=0 to a circuit contianing a capacitance C and inductance L. The current i satisfies the equation $L\frac{di}{dt} + \frac{1}{C}\int i\, dt = E_0 \sin pt$. If $p^2 = 1/LC$ and initially the current i and the charge q are zero. Find the current i any time t in the circuit (5)

(b) Solve the equation $y = 2px - p^2$, Where $P = \frac{dy}{dx}$. (3)

Q5. (a) Solve the differential equation $xy(1+xy^2)\frac{dy}{dx} = 1$ (4)

(b) Find the solution of the differential equation $(xy^3+y)dx + 2(x^2y^2+x+y^4)dy = 0$ (4)

SECTION-C

Q6. (a) Use the rank method to test the consistency of the system of equations (4)

$$x+2y-2z=1;\ 2x-3y+z=0;\ 5x+y-5z=1;\ 3x+14y-12z=5,$$

if consistent, then solve it completely.

(b) State Cayley-Hamilton theorem and use it to find the inverse of the matrix (4)

$A=\begin{bmatrix}1 & 2 & 0\\ -1 & 1 & 2\\ 1 & 2 & 1\end{bmatrix}$.

Q7. (a) Test for what values of x the series (4)

$\frac{1}{2}+\frac{2}{3}x+\left(\frac{3}{4}\right)^2 x^2+\left(\frac{4}{5}\right)^3 x^3+\ldots\ldots\ldots\infty,(x>0)$ Converges /diverges.

(b) Test for the convergence/diverge of the following series

(i) $\sum_{n=1}^{\infty}\text{Sin}\frac{1}{n}$ (ii) $\sum_{n=1}^{\infty}\frac{1.3.5......(2n-1)}{4^n 2^n (n!)}$ (4)

Q8. (a) Use Demoivre's theorem to prove that (4)

$$(1+\sin\theta+i\cos\theta)^n+(1+\sin\theta-i\cos\theta)^n=2^{n+1}\cos^n\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\cos\left(\frac{n\pi}{4}-\frac{n\theta}{2}\right)$$

(b) Seperate $\text{Sin}^{-1}(e^{-i\theta})$ into real and imaginary parts, Where θ is a positive acute angle. (4)

Q9. (a) Find the sum of the series (5)

$$1-\frac{1}{2}\cos\theta+\frac{1.3}{2.4}\cos 2\theta-\frac{1.3.5}{2.4.6}\cos 3\theta+\ldots\ldots\ldots,\ -\pi<\theta<\pi.$$

(b) Reduce the matrix $\begin{bmatrix}1 & 1 & 2\\ 1 & 2 & 3\\ 0 & -1 & -1\end{bmatrix}$ to normal form and hence find the rank of the matrix. (3)

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